Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•1 evidence use of quotient rule with denominator and one term of numerator correct [1]	•1 $\dfrac{-3\left(x^2+4\right)\ldots}{\left(x^2+4\right)^2}$ **OR** $\dfrac{\ldots-2x(1-3x)}{\left(x^2+4\right)^2}$	**3**
			•2 complete differentiation	•2 $\dfrac{-3\left(x^2+4\right)-2x(1-3x)}{\left(x^2+4\right)^2}$	
			•3 simplify [2,3]	•3 $\dfrac{3x^2-2x-12}{\left(x^2+4\right)^2}$	
Notes: 1. Where a candidate equates $\dfrac{dy}{dx}$ to y do not withhold •1. 2. •3 is available only where candidates have multiplied out brackets and collected like terms on the numerator. 3. Do not award •3 for incorrect manipulation subsequent to a correct answer.					
Commonly Observed Response: Candidates who use the product rule: •1 $-3\left(x^2+4\right)^{-1}+(1-3x)\ldots$ or $\ldots\left(x^2+4\right)^{-1}+(1-3x)-2x\left(x^2+4\right)^{-2}$ •2 $-3\left(x^2+4\right)^{-1}-2x(1-3x)\left(x^2+4\right)^{-2}$ •3 $\left(3x^2-2x-12\right)\left(x^2+4\right)^{-2}$					
	(b)		•4 start differentiation [1]	•4 $-\text{cosec}5x\cot 5x$	**2**
			•5 apply chain rule	•5 $-5\text{cosec}5x\cot 5x$	
Notes: **1.** Where a candidate equates $f'(x)$ to $f(x)$, •4 is not available. See COR.					
Commonly Observed Responses: $f(x)=\text{cosec}5x$ $=-5\text{cosec}5x\cot 5x$ Unless subsequently corrected, award •5 only.					

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ set up augmented matrix	•¹ $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 2 & 1 & -3 & 3 \\ 1 & -7 & -4 & 9 \end{array}\right)$	**4**
			•² obtain two leading zeros [1]	•² $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -5 & -5 \\ 0 & -5 & -5 & 5 \end{array}\right)$	
			•³ complete row operations [1]	•³ $\left(\begin{array}{ccc\|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -5 & -5 \\ 0 & 0 & -10 & 0 \end{array}\right)$	
			•⁴ obtain solution [2]	•⁴ $x=2,\ y=-1,\ z=0$	

Notes:

1. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •² and •³.
2. Where a candidate erroneously obtains inconsistency or redundancy in the system and communicates this, •⁴ is available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ find conjugate	•¹ $6-2i$	**2**
			•² multiply [1]	•² $36+8i$	

Notes:

1. Where a candidate does not identify $\overline{z_2}$, •² is still available for any correct expansion and simplification of the form $(a+bi)(c+di)$.

Commonly Observed Responses:

$(5+3i)(6+2i)$ leading to $24+28i$ award 1/2.

$\frac{5+3i}{6+2i}\times\frac{6-2i}{6-2i}$ leading to $\frac{9+2i}{10}$ award 1/2.

$\frac{5+3i}{6-2i}\times\frac{6+2i}{6+2i}$ leading to $\frac{6+7i}{10}$ award 1/2.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ apply product or chain rule	•¹ $3y^2\frac{dy}{dx}$ or $2y+2x\frac{dy}{dx}$	**3**
			•² complete differentiation	•² $3y^2\frac{dy}{dx}+4\frac{dy}{dx}=2y+2x\frac{dy}{dx}$	
			•³ find expression for $\frac{dy}{dx}$ [1]	•³ $\frac{dy}{dx}=\frac{2y}{3y^2+4-2x}$	

Notes:

1. •³ is available only where $\frac{dy}{dx}$ appears more than once after the candidate has completed their differentiation.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁴ evaluate gradient [1, 2]	•⁴ $m=-2$	**1**

Notes:

1. For •⁴, accept $\frac{dy}{dx}=-2$.
2. •⁴ is available only if a candidate's expression for $\frac{dy}{dx}$ contains both x and y terms.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•⁵ find value of y [1, 2]	•⁵ $y=0$	**2**
			•⁶ demonstrate inconsistency [3]	•⁶ LHS $=0$, RHS $=1$ $\therefore$ inconsistent	

Notes:

1. •⁵ is available only where $\frac{dy}{dx}$ is a fraction where the numerator and denominator both contain a variable.
2. •⁵ is not available when $y=0$ appears as part of an incorrect assumption eg 'stationary points when $x=0$, $y=0$'.
3. Working for •⁶ must include substitution of $y=0$ and retention of the variable x in the original equation.

Commonly Observed Responses:

Communication at •⁶

Working leading to '$0=1$' with no further communication. Do not award •⁶.

Working leading to '$0=1$ no stationary point'. Award •⁶.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		**Method 1**	**Method 1**	2
				$f(x)=e^{-4x}$ $\quad f(0)=1$	
			•1 all three derivatives AND all four evaluations	•1 $f'(x)=-4e^{-4x}$ $\quad f'(0)=-4$ $f''(x)=16e^{-4x}$ $\quad f''(0)=16$ $f'''(x)=-64e^{-4x}$ $\quad f'''(0)=-64$ stated or implied	
			•2 obtain simplified expression [1, 2]	•2 $1-4x+8x^2-\frac{32}{3}x^3$	
			Method 2	**Method 2**	
			•1 write down Maclaurin series for e^x	•1 $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$ stated or implied	
			•2 substitute and simplify [1, 2]	•2 $1-4x+8x^2-\frac{32}{3}x^3$	

Notes:

1. The simplification for •2 may appear in (b).
2. Do not accept $+-4x$ or $+-\frac{32}{3}x^3$ at •2 unless simplified in (b). Do accept $+\frac{-32}{3}x^3$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•[3] express as a product [1] •[4] expand and simplify [1, 2]	•[3] $(3+2x)e^{-4x}$ stated or implied •[4] $3-10x+16x^2-16x^3$	2

Notes:

1. For candidates who produce an incorrect Maclaurin expansion for $3+2x$ and multiply this by e^{-4x}, •[3] may be awarded but •[4] is unavailable.
2. At •[4], ignore any higher order terms.

Commonly Observed Responses:

Applying Maclaurin expansion to quotient

•[3] first derivative and two evaluations OR all three derivatives. Derivatives need not be simplified.

•[4] complete expansion.

$f(x)=\dfrac{3+2x}{e^{4x}}$ $\quad\quad$ $f(0)=3$

$f'(x)=\dfrac{-10-8x}{e^{4x}}$ $\quad\quad$ $f'(0)=-10$

$f''(x)=\dfrac{32+32x}{e^{4x}}$ $\quad\quad$ $f''(0)=32$

$f'''(x)=\dfrac{-96-128x}{e^{4x}}$ $\quad\quad$ $f'''(0)=-96$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		$\bullet^1$ find counterexample AND state composite number AND communicate [1, 2]	$\bullet^1$ eg when $n = 9$ AND $n^2 + 4 = 85$ AND which is not prime	1

Notes:

1. Acceptable communication to demonstrate that a number is not prime at $\bullet^1$ may take the form of eg:
   - 85 is divisible by 5
   - $85 = 5 \times 17$
   - 85 is composite
2. Insufficient communication includes '85 so false'.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^2$ correct form for two consecutive integers [1, 2, 3]	$\bullet^2$ eg n, $n+1$ AND $n \in Z$	3
			$\bullet^3$ correct expression [1, 4]	$\bullet^3$ $(n+1)^3 - n^3$	
			$\bullet^4$ multiply, express in correct form and communicate [1, 5, 6, 7]	$\bullet^4$ eg $3(n^2+n)+1$ which is not divisible by 3	

Notes:

1. Where a candidate uses two unrelated variables, eg n and $x+1$, award 0/3.
2. At $\bullet^2$, accept $n \in Z$ expressed in words.
3. Where a candidate does not consider every pair of consecutive integers, eg $2k$ and $2k+1$, $\bullet^2$ is not available.
4. For the award of $\bullet^3$ at least one expression must be of the form $(an \pm b)^3$ where $a, b \neq 0$.
5. $\bullet^4$ is available only where a candidate has processed two cubed expressions.
6. Reference to 'false' at $\bullet^4$ is acceptable only if it references divisibility by 3 and not the statement.
7. At $\bullet^4$, after taking a common factor of 3 to leave a remainder, acceptable communication may take the form of eg
   - … therefore true
   - … gives a remainder of 1

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•1 differentiate [1]	•1 $\frac{du}{dy} = 2y$ or $du = 2y\,dy$	4
			•2 determine new limits and begin to rewrite integral [1]	•2 $\int_1^{26} \ldots du$	
			•3 complete integral [2]	•3 $2\int_1^{26} \frac{1}{\sqrt{u}}\,du$	
			•4 integrate and evaluate [3]	•4 $4\left(\sqrt{26}-1\right)$	

Notes:

1. Where a candidate attempts to process an integrand containing both u and y, only •1 and •2 are available.
2. Limits are not necessary for the award of •3.
3. Where a candidate erroneously includes a limit of 0 for u, •4 is not available.

Commonly Observed Responses:

A No limits in new integral and return to original variable:

$2\int \frac{1}{\sqrt{u}}\,du$ award •3

$\left[4(y^2+1)^{\frac{1}{2}}\right]_0^5$ award •2

B Wrong limits in new integral and return to original variable:

$\int_0^5 \ldots du$ do not award •2

$2\int_0^5 \frac{1}{\sqrt{u}}\,du$ award •3

$\left[4(y^2+1)^{\frac{1}{2}}\right]_0^5$ leading to $4\left(\sqrt{26}-1\right)$ award •4

C Wrong limits in new integral and no return to original variable:

$\int_0^5 \ldots du$ do not award •2

$2\int_0^5 \frac{1}{\sqrt{u}}\,du$ award •3

$\left[4u^{\frac{1}{2}}\right]_0^5$ do not award •4

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		$\bullet^5$ obtain area [1]	$\bullet^5$ $8\left(\sqrt{26}-1\right)$ (square units)	**1**

Notes:

1. For a candidate who obtains a negative value at $\bullet^4$, $\bullet^5$ is available only where they interpret the area as a positive number.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		$\bullet^6$ obtain expression [1]	$\bullet^6$ $1-\dfrac{1}{y^2+1}$	**1**

Notes:

1. At $\bullet^6$, accept $1+\dfrac{-1}{y^2+1}$ but do not accept $1+-\dfrac{1}{y^2+1}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(d)		•⁷ correct form of integral [1, 2]	•⁷ $\pi\int_0^5 x^2\,dy$	**4**
			•⁸ find expression in integrable form	•⁸ $16\pi\int_0^5\left(1-\frac{1}{y^2+1}\right)dy$	
			•⁹ integrate $-\frac{1}{y^2+1}$ [3, 4]	•⁹ $\ldots-\tan^{-1}y$	
			•¹⁰ complete integration and evaluate [3, 5]	•¹⁰ $16\pi\left(5-\tan^{-1}5\right)$ (cubic units)	

Notes:

1. For the award of •⁷:
   - limits must appear at some point
   - dy must appear at some point.
2. At •⁷ accept $V=\pi\int_0^5\left[f(y)\right]^2 dy$.
3. Where a candidate attempts to process an integrand containing both u and y, •⁹ and •¹⁰ are unavailable.
4. •⁹ is available only for the integration of a term of the form $\pm\frac{b}{y^2+1}$, $b \in$ □.
5. •¹⁰ is not available where:
   - the integrand at •⁸ is not beyond Higher Mathematics standard
   - a candidate produces an undefined expression eg $\ln 0$.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ state expression •² form equation **AND** obtain one of *A*, *B* and *C* •³ state expression 1	•¹ $\frac{A}{x}+\frac{Bx+C}{x^2+5}$ •² $3x^2-3x+5=A\left(x^2+5\right)+\left(Bx+C\right)x$ **and** any one from $A=1,\ B=2,\ C=-3$ •³ $\frac{1}{x}+\frac{2x-3}{x^2+5}$	3

Notes:

1. At •³ accept values explicitly stated for A, B and C provided the template is written at •¹.

Commonly Observed Responses:

Two constant numerators

For $\frac{3x^2-3x+5}{x\left(x^2+5\right)}=\frac{A}{x}+\frac{B}{x^2+5}$ award 0/3.

One constant and one incorrect linear numerator

For $\frac{3x^2-3x+5}{x\left(x^2+5\right)}=\frac{A}{x}+\frac{Bx}{x^2+5}$ award 0/3.

One constant and one quadratic numerator

•¹ $\frac{A}{x}+\frac{Bx^2+Cx+D}{x^2+5}$

•² $3x^2-3x+5=A\left(x^2+5\right)+\left(Bx^2+Cx+D\right)x$ AND $B=0$ and one other from $A=1, C=2, D=-3$

•³ $\frac{1}{x}+\frac{2x-3}{x^2+5}$

Where a candidate uses this approach but does not find $B=0$, •² and •³ are unavailable.

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ begin integration [1]	•¹ $\ln(2x+1)$	2
			•² complete integration and evaluate [2, 3]	•² $2\ln 7$	

Notes:

1. Where a candidate uses a substitution of eg $u = 2x+1$, award •¹ for the appearance of $\ln u$.
2. At •² do not accept an unsimplified final answer of $2(\ln 7 - \ln 1)$.
3. At •² accept $2\ln|7|$ or $\ln 7^2$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.				$634 = 7 \times 87 + 25$	3
			•1 start process and obtain remainder of 1	•1 $87 = 3 \times 25 + 12$ $25 = 2 \times 12 + 1$	
			•2 express gcd in terms of 87 and 25	•2 $1 = 25 - 2(87 - 3 \times 25)$	
			•3 obtain a and b [1]	•3 $a = 7 \quad b = -51$	

Notes:

1. At •3, where candidates do not explicitly communicate a and b, accept eg $634 \times 7 + 87 \times -51 = 1$ or $634 \times 7 + -51 \times 87 = 1$. Do not accept $634 \times 7 - 87 \times 51 = 1$ or $634 \times 7 - 51 \times 87 = 1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] integrate to find "$uv-$" [1] •[2] differentiate to find "$\int u'v\,dx$" [2] •[3] obtain full solution [3, 4]	•[1] $\frac{1}{3}(x+2)(2x+7)^{\frac{3}{2}}-\ldots$ •[2] $\ldots\int\frac{1}{3}(2x+7)^{\frac{3}{2}}\,dx$ •[3] $\frac{1}{3}(x+2)(2x+7)^{\frac{3}{2}}-\frac{1}{15}(2x+7)^{\frac{5}{2}}+c$	3

Notes:

1. For candidates who choose to integrate $(x+2)$ and differentiate $(2x+7)^{\frac{1}{2}}$ and process this correctly award •[1] only.
2. Disregard the omission of dx at •[2].
3. General marking principle (l) applies at •[3] to candidates who eg repeatedly fail to divide by 2.
4. Disregard omission of $+c$ at •[3].

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ begin process	•¹ $1\begin{vmatrix} k & 3 \\ 18 & -7 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ k & -7 \end{vmatrix} + 1\begin{vmatrix} 2 & k \\ k & 18 \end{vmatrix}$ or equivalent	**3**
			•² find determinant	•² $-k^2+2k+24$ stated or implied	
			•³ equate to zero and solve for k [1, 2]	•³ eg $(k-6)=0$, $(k+4)=0$ leading to 6 and -4	

Notes:

1. •³ may be awarded only if '= 0' appears at some point in the solution.

2. •³ may be awarded only where $k \in$ □ .

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] use two terms to find difference [1, 2, 3] •[2] check with a different pair of terms and conclude [1, 3, 4]	•[1] eg $(3x+2)-(x+5)=2x-3$ •[2] eg $(5x-1)-(3x+2)=2x-3$, common difference	2

Notes:

1. Where a candidate picks a numerical value for x, •[1] and •[2] are not available unless an algebraic approach is also considered in (a) or $2x-3$ appears in (b) or (c).
2. Award •[1] for $2x-3$ with no working.
3. Accept $3-2x$ for the award of •[1] and in subsequent working for •[2].
4. For •[2] candidates must show algebraically a link between two differences and communicate commonality. See CORS.

Commonly Observed Responses:

'$x+5 \underbrace{\quad}_{} 3x+2 \quad 5x-1$' — with $2x-3$ written above each gap: '$x+5 \overset{2x-3}{\quad} 3x+2 \overset{2x-3}{\quad} 5x-1$ so there is a common difference' award 2/2.

Working eg $\begin{array}{l} 3x+2-(x+5)=2x-3 \\ 3x+2+2x-3=5x-1 \end{array}$ followed by communication:

- 'so arithmetic' award 2/2
- 'same difference' award 2/2
- 'difference is $2x-3$' award 2/2
- 'difference' award 1/2. Insufficient communication.

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] substitute into formula [1, 2, 3, 4] •[4] simplify expression [1, 4]	•[3] $x+5+(15-1)(2x-3)$ •[4] $29x-37$	2

Notes:

1. Where a candidate substitutes a numerical value for either x or d, •[3] and •[4] are unavailable unless an algebraic approach is also present.
2. Where a candidate uses $3-2x$ from (a), •[3] is unavailable.
3. Where a candidate substitutes an incorrect algebraic expression for d, other than $3-2x$, •[3] is available only if they have identified the expression as the common difference in (a), either in words or "$d=\ldots$".
4. When an incorrect algebraic expression appears at •[3], •[4] may be awarded only where the candidate has had to expand brackets and collect like terms.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[5] substitute into formula and equate [1, 2, 3] •[6] solve for x [1, 4]	•[5] $\frac{20}{2}\left[2(x+5)+(20-1)(2x-3)\right]=1130$ •[6] $x=4$	**2**

Notes:

1. Where a candidate substitutes a numerical value for x or d, •[5] and •[6] are unavailable unless an algebraic approach is also present.
2. Where a candidate substitutes $3-2x$ for d, •[5] is available only if $3-2x$ appears as part of an expression at •[3].
3. Where a candidate substitutes an incorrect algebraic expression for d, other than $3-2x$, •[5] is available only if they have identified the expression as the common difference in (a) or (b),

- in words
- by writing "$d = \ldots$"
- as an entry for a common difference in the correct expression at (b)
- as an entry identified as d in an incorrect expression at (b).

4. •[6] is available only where $x \in \square$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ state second root	•¹ $3-i$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² form product of linear factors •³ find the value of a [1, 2]	•² $\left(z-(3-i)\right)\left(z-(3+i)\right)$ •³ 10	**2**

Notes:

1. For the award of •³, $a \in \square$.
2. **Beware** of incorrect working leading to $a=10$ in •³. Where a candidate produces $z^2-6z+10$ and proceeds to use algebraic division, •³ is not available for any remainder other than zero or $a-10$.

Commonly Observed Responses:

Alternative strategies for •²
Multiply roots
$(3+i)(3-i)$

Substitute either root into the quadratic
eg $(3+i)^2-6(3+i)+a=0$

Start process for synthetic division
eg as far as

$$\begin{array}{r|ccc} 3+i & 1 & -6 & a \\ & & 3+i & \\ \hline & 1 & -3+i & \end{array}$$

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•⁴ find the value of b [1, 2, 3]	•⁴ 50	**1**

Notes:

1. For the award of •⁴, $b \in \square$.
2. For an answer with no working or insufficient working award 0/1.
3. **Beware** of incorrect working leading to $b=50$ in •⁴. Where a candidate uses algebraic division, •⁴ is not available for any remainder other than zero or $b-50$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] evidence of use of product rule and one term correct [1] •[2] complete differentiation [1]	•[1] $\ln x + \ldots$ **OR** $\ldots + x \times \frac{1}{x}$ •[2] $\ln x$	**2**

Notes:

1. For an answer with no working award 0/2.

Commonly Observed Responses:

	(b)		•[3] state form of integrating factor [1] •[4] find integrating factor [1,2] •[5] rewrite as integral equation [1,2,3,4] •[6] integrate and rewrite [5,6]	•[3] $e^{\int \ln x\, dx}$ stated or implied •[4] $e^{x\ln x - x}$ •[5] $ye^{x\ln x - x} = \int e^{x\ln x - x} x^{-x}\, dx$ •[6] $y = \frac{-e^{-x} + c}{x^x e^{-x}}$	**4**

Notes:

1. For any attempt to separate variables, award 0/4.
2. •[5] is available for candidates who either have an incorrect integrating factor at •[4] or who have incorrectly simplified a correct integrating factor prior to •[5]. SEE CORS.
3. Do not withhold •[5] for the omission of 'dx' on the right-hand side.
4. Where a candidate writes $\int \frac{d}{dx} \ldots$ on the left-hand side do not withhold •[5] provided the candidate later produces evidence that they have integrated with respect to x.
5. Accept $y = \frac{-e^{-x} + c}{e^{x\ln x - x}}$ for the award of •[6].
6. For a candidate who omits the constant of integration or produces an integrand which is not beyond Higher Mathematics standard, •[6] is not available.

Commonly Observed Responses:

Correct integrating factor at •[4] followed by incorrect simplification

$e^{\int \ln x\, dx}$ leading to eg $e^{x\ln x - x} = \frac{1}{x}$. Award •[3] and •[4]. If followed by $y\frac{1}{x} = \int \frac{1}{x} x^{-x}\, dx$, award •[5]. Do not award •[6].

Incorrect integrating factor at •[4]

$e^{\int \ln x\, dx}$ leading to eg $\frac{1}{x}$ (without evidence of $e^{x\ln x - x}$). Award •[3], do not award •[4]. If followed by $y\frac{1}{x} = \int \frac{1}{x} x^{-x}\, dx$, award •[5].

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 show true for $n=1$ [1]	•1 when $n=1$ $\begin{pmatrix} 3 & 1-3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$ so true for $n=1$.	5
			•2 assume true for $n=k$ and consider $n=k+1$ [2]	•2 assume true for $n=k$ AND $A^k = \begin{pmatrix} 3^k & 1-3^k \\ 0 & 1 \end{pmatrix}$ AND $A^{k+1} = \ldots$	
			•3 use inductive hypothesis	•3 $\begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 1-3^k \\ 0 & 1 \end{pmatrix}$	
			•4 multiply matrices [3]	•4 $\begin{pmatrix} 3\times 3^k & 3\times\left(1-3^k\right)+\left(-2\right) \\ 0 & 1 \end{pmatrix}$	
			•5 simplify to appropriate form and communicate result [4, 5, 6]	•5 $\begin{pmatrix} 3^{k+1} & 3-3^{k+1}-2 \\ 0 & 1 \end{pmatrix}$ leading to $\begin{pmatrix} 3^{k+1} & 1-3^{k+1} \\ 0 & 1 \end{pmatrix}$ so if result is true for $n=k$ then it is true for $n=k+1$. But it is true for $n=1$ and so it is true $\forall n \in N$ by mathematical induction.	

Question		Generic scheme	Illustrative scheme	Max mark
9.	(continued)			

Notes:

1. ‘ $A=\begin{pmatrix}3 & -2\\ 0 & 1\end{pmatrix}$, $A^1=\begin{pmatrix}3 & -2\\ 0 & 1\end{pmatrix}$’ and/or ‘True for n =1’ are insufficient for the award of •¹ . A candidate must demonstrate evidence of substitution.

2. For •² sufficient phrases for $n=k$ contain:

   - ‘If true for...’; ‘Suppose true for…’; ‘Assume true for…’.

   For •² insufficient phrases for $n=k$ contain:

   - ‘Consider $n=k$’, ‘assume $n=k$’, ‘assume $n=k$ is true’ and ‘True for $n=k$’.

   A sufficient phrase for the award of •² may appear at •⁵.

   For •², accept:

   - assume true for $n=k$ AND $A^k=\begin{pmatrix}3^k & 1-3^k\\ 0 & 1\end{pmatrix}$ AND ‘Aim/goal: $A^{k+1}=\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\ 0 & 1\end{pmatrix}$’.

   For •² unacceptable phrases for $n=k+1$ contain:

   - ‘Consider true for $n=k+1$’, ‘true for $n=k+1$’;
   - ‘ $A^{k+1}=\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\ 0 & 1\end{pmatrix}$’ (with no reference to aim/goal or no further processing) .

3. Award •⁴ where products resulting in zero are shown.
4. •⁵ is unavailable to candidates who write down the correct expression without algebraic justification.
5. Full marks are available to candidates who state an aim/goal earlier in the proof and who subsequently reference the stated aim/goal.
6. Following the required algebra and statement of the inductive hypothesis, the minimum acceptable response for •⁵ is ‘Then true for $n=k+1$, but since true for $n=1$, then true for all n’ or equivalent.

Commonly Observed Responses:

Multiplication order A^kA

•³ $\begin{pmatrix}3^k & 1-3^k\\ 0 & 1\end{pmatrix}\begin{pmatrix}3 & -2\\ 0 & 1\end{pmatrix}$

•⁴ $\begin{pmatrix}3^k\times 3 & 3^k\times(-2)+1-3^k\\ 0 & 1\end{pmatrix}$ stated or implied by

•⁵ $\begin{pmatrix}3^{k+1} & 3^k\times(-2)+1-3^k\\ 0 & 1\end{pmatrix}$ leading to $\begin{pmatrix}3^{k+1} & 1-3^{k+1}\\ 0 & 1\end{pmatrix}$

so if result is true for $n=k$ then it is true for $n=k+1$. But it is true for $n=1$ and so it is true $\forall n\in N$ by mathematical induction.

Question			Generic scheme	Illustrative scheme	Max mark
10.			•¹ construct auxiliary equation [1,4,5]	•¹ $m^2 - 4m + 4 = 0$	**9**
			•² find complementary function [2,3,4,]	•² $y = Ae^{2x} + Bxe^{2x}$	
			•³ state particular integral and obtain first and second derivatives of particular integral	•³ $y = C\sin x + D\cos x$ $\dfrac{dy}{dx} = C\cos x - D\sin x$ $\dfrac{d^2y}{dx^2} = -C\sin x - D\cos x$	
			•⁴ substitute into LHS of differential equation [6]	•⁴ $(-C\sin x - D\cos x)$ $-4(C\cos x - D\sin x)$ $+4(C\sin x + D\cos x)$	
			•⁵ derive simultaneous equations [6]	•⁵ $3C + 4D = 9$ $-4C + 3D = 13$	
			•⁶ obtain both constants of particular integral [6]	•⁶ $C = -1 \quad D = 3$	
			•⁷ differentiate general solution [6,7]	•⁷ $\dfrac{dy}{dx} = 2Ae^{2x} + Be^{2x} + 2Bxe^{2x}$ $-\cos x - 3\sin x$	
			•⁸ find coefficient of e^{2x} [4,5,6]	•⁸ $A = 2$	
			•⁹ find second coefficient and state particular solution [3,5,6,8]	•⁹ $B = -3$ $y = 2e^{2x} - 3xe^{2x} - \sin x + 3\cos x$	

Question		Generic scheme	Illustrative scheme	Max mark
10.	(continued)			

Notes:

1. $\bullet^1$ is not available where $'=0'$ has been omitted.
2. $\bullet^2$ may still be awarded if the complementary function appears only as part of a general solution or the particular solution.
3. Do not withhold $\bullet^2$ for the omission of $'y=...'$ provided it appears as part of a general solution or at $\bullet^9$.
4. Where a candidate does not introduce a particular integral only $\bullet^1$, $\bullet^2$ and $\bullet^8$ are available.
5. Where a candidate introduces a particular integral which does not involve any trigonometric terms, only $\bullet^1$, $\bullet^2$, $\bullet^8$ and $\bullet^9$ are available.
6. Where a candidate introduces a particular integral with only one trigonometric term $\bullet^4$ is available but $\bullet^5$ and $\bullet^7$ are not. Moreover,
   - award $\bullet^6$ for communicating inconsistency when comparing coefficients eg '$3C=9$ and $C=0$ - not possible'. Proceeding with a non-zero value for eg C, $\bullet^8$ and $\bullet^9$ are also available.
   - only $\bullet^8$ and $\bullet^9$ are available when inconsistency is not recognised.
7. $\bullet^7$ is available only where differentiation involves:
   - a complementary function requiring the product rule AND
   - a particular integral with both $\sin x$ and $\cos x$.
8. $\bullet^9$ is available only where a (correct or incorrect) particular integral which is consistent with previous working (and is not simply appearing for the first time at this stage) is included as part of the particular solution.

Commonly Observed Responses:

Auxiliary Equation Correct

$m^2-4m+4=0$

Complementary function of $y=Ae^{-2x}+Bxe^{-2x}$ (leading to particular solution of $y=2e^{-2x}+5xe^{-2x}-\sin x+3\cos x$) do not award $\bullet^2$.

Auxiliary Equation Incorrect

$m^2+4m+4=0$

Complementary function of $y=Ae^{-2x}+Bxe^{-2x}$ (leading to particular solution of $y=2e^{-2x}+5xe^{-2x}-\sin x+3\cos x$) do not award $\bullet^1$.

Complementary Function Incorrect

Complementary function of eg $y=Ae^{2x}+Be^{2x}$ (leading to inconsistent values of A and B), $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available.

Question			Generic scheme	Illustrative scheme	Max mark
11.			$\bullet^1$ find $\frac{dx}{dt}$	$\bullet^1$ $\frac{dx}{dt} = \frac{2}{1+(2t)^2}$	**4**
			$\bullet^2$ equate expressions for $\frac{dy}{dx}$ 1	$\bullet^2$ $\dfrac{\frac{dy}{dt}}{\left(\frac{2}{1+4t^2}\right)} = 6t\left(1+4t^2\right)$	
			$\bullet^3$ simplified expression for $\frac{dy}{dt}$ 1, 2, 3	$\bullet^3$ $\frac{dy}{dt} = 12t$	
			$\bullet^4$ find expression for y 1, 2, 3	$\bullet^4$ $y = 6t^2 - 1$	

Notes:

1. For candidates who attempt to find $\int 6t\left(1+4t^2\right)dt$, $\bullet^2$, $\bullet^3$ and $\bullet^4$ are unavailable.
2. $\bullet^3$ and $\bullet^4$ are available only where a candidate clearly identifies an expression for $\frac{dy}{dt}$.
3. $\bullet^4$ is available where a candidate produces a non-zero constant term for $\frac{dy}{dt}$ at $\bullet^3$.

Commonly Observed Responses:

Candidates who do not apply the chain rule at $\bullet^1$.

$\frac{dx}{dt} = \frac{1}{1+(2t)^2}$ do not award $\bullet^1$

$\bullet^2$ $\dfrac{\frac{dy}{dt}}{\left(\frac{1}{1+4t^2}\right)} = 6t\left(1+4t^2\right)$

$\bullet^3$ $\frac{dy}{dt} = 6t$

$\bullet^4$ $y = 3t^2 + 2$

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•1 expression	•1 $\cos 4\theta + i\sin 4\theta$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•2 binomial expansion [1, 2, 3]	•2 $(\cos\theta)^4 + 4(\cos\theta)^3(i\sin\theta)$ $+6(\cos\theta)^2(i\sin\theta)^2 + 4(\cos\theta)(i\sin\theta)^3$ $+(i\sin\theta)^4$	**3**
			•3 simplify three terms [1, 2, 3]	•3 three from: $\cos^4\theta + 4\cos^3\theta i\sin\theta - 6\cos^2\theta\sin^2\theta$ $-4\cos\theta i\sin^3\theta + \sin^4\theta$	
			•4 complete simplification [2, 3]	•4 $\cos^4\theta + 4\cos^3\theta i\sin\theta - 6\cos^2\theta\sin^2\theta$ $-4\cos\theta i\sin^3\theta + \sin^4\theta$	

Notes:

1. Do not withhold •2 for the omission of brackets provided there is evidence to award •3 .
2. Where i terms are not raised to a power greater than one and are not later resolved, •2 •3 •4 are not available.
3. Where a candidate does not write out a full binomial expansion, •2 is unavailable but •3 and •4 may still be awarded.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(c)	(i)	$\bullet^5$ equate real parts [1] $\bullet^6$ obtain result [1]	$\bullet^5$ $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ $\bullet^6$ $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\left(1-\cos^2\theta\right) + \left(1-\cos^2\theta\right)^2$ leading to $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	2

Notes:

1. Do not withhold $\bullet^5$ for the omission of '$\cos 4\theta =$' provided it appears at $\bullet^6$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	$\bullet^7$ expression for $\cot 4\theta$ in terms of $\sin\theta$ and $\cos\theta$ [1] $\bullet^8$ rewrite in terms of $\cos\theta$ [2, 3]	$\bullet^7$ $\dfrac{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}$ $\bullet^8$ $\sin\theta\cot 4\theta = \dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{8\cos^3\theta - 4\cos\theta}$	2

Notes:

1. Accept $\dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}$ for the award of $\bullet^7$.
2. Accept $\sin\theta\cot 4\theta = \dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{4\cos^3\theta - 4\cos\theta\left(1-\cos^2\theta\right)}$ for the award of $\bullet^8$.
3. Do not withhold $\bullet^8$ for the omission of $\sin\theta\cot 4\theta$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)	(i)	•[1] full turn divided by time [1, 2]	•[1] $\frac{2\pi}{12}$ leading to $\frac{\pi}{6}$.	1

Notes:

1. •[1] is available to candidates who work in degrees and then convert to radians.
2. If $\frac{2\pi}{12}$ follows directly from an incorrect assumption, eg the rate of change of LP with respect to time is a constant, then •[1] is not available.

Commonly Observed Responses:

Angular velocity approach

Given that the angular velocity in this case is constant, •[1] may be awarded.

$$\begin{aligned}\omega &= \frac{\theta}{t}\\ &= \frac{d\theta}{dt}\\ &= \frac{2\pi}{12}\\ &= \frac{\pi}{6}\end{aligned}$$

Question			Generic scheme	Illustrative scheme	Max mark
		(ii)	•[2] state relationship between derivatives	•[2] $\frac{dx}{dt}=\frac{dx}{d\theta}\cdot\frac{d\theta}{dt}$	4
			•[3] find x in terms of θ	•[3] $x=10\tan\theta$	
			•[4] differentiate wrt θ [1, 2]	•[4] $\frac{dx}{d\theta}=10\sec^2\theta$ stated or implied	
			•[5] substitute and simplify	•[5] $\frac{dx}{dt}=10\sec^2\theta.\frac{\pi}{6}$ leading to $\frac{dx}{dt}=\frac{5\pi}{3}\sec^2\theta$	

Notes:

1. •[4] is not available for a candidate who differentiates $x=\frac{5\pi}{3}\tan\theta$.
2. •[4] may be awarded only where a candidate differentiates a clearly defined expression for x, other than $x=\frac{5\pi}{3}\tan\theta$, and the differentiation is beyond Higher Mathematics standard.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•⁶ complete proof [1]	•⁶ eg $1+\dfrac{\sin^2\theta}{\cos^2\theta}$ $=\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ or $\dfrac{\cos^2\theta}{\cos^2\theta}+\dfrac{\sin^2\theta}{\cos^2\theta}$ $=\dfrac{1}{\cos^2\theta}$ $=\sec^2\theta$	**1**

Notes:

1. Accept proof using another (single) variable in place of θ.

Commonly Observed Responses:

Starting with $\cos^2\theta+\sin^2\theta=1$

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos\ \ \theta}=\frac{1}{\cos^2\theta}$$

$$1+\tan^2 2\theta=\sec^2\theta$$

Both lines of working must be present for the award of •⁶

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•⁷ find $\tan\theta$ or $\sec\theta$	•⁷ $\tan\theta=\dfrac{5}{10}$ or $\sec\theta=\dfrac{\sqrt{125}}{10}$	**3**
			•⁸ corresponding value of $\sec^2\theta$ [1]	•⁸ $1+\left(\dfrac{5}{10}\right)^2$, $\dfrac{125}{100}$ or $1+\left(\tan\left(\tan^{-1}\dfrac{1}{2}\right)\right)^2$ or equivalent	
			•⁹ substitute and simplify [2, 3]	•⁹ $\dfrac{25\pi}{12}$ ms^{-1}.	

Notes:

1. Where a candidate finds and rounds an angle for θ, •⁸ is available only when rounding occurs to at least 2 significant figures.
2. Where a candidate substitutes an angle in degrees into their calculation, withhold •⁹.
3. Units must appear beside an exact value for the award of •⁹.

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]